In conclusion, we discuss a possible practical application of the theoretical results obtained here. The excitation of surface waves by beams of nonrelativistic electrons in a cold plasma in the presence of a variable electric field is investigated. In certain conditions the interaction of the electron beam with plasma leads to the excitation of unstable oscillations when initially small perturbations of density and particle velocities, the amplitudes of the self-consistent electric field in the plasma, and so forth, exponentially increase with time (or with the increase of the spatial coordinate). However, a need arises for the suppression of the beam instability; this occurs in the operation of experimental equipment in which electron beams exist. Among such devices we can mention discharges with longitudinal electric field (for example, installations of "tokamak" type). On the periphery of the plasma configuration the density of the plasma particles is considerably smaller than the plasma density at the axis of the system. For example, in the case of a sufficiently rapid process when skinning of the current occurs, the plasma density has a sharp discontinuity at the boundary of the discharge. Under the action of the rotating electric field the plasma electrons in the peripheral region of the discharge can get accelerated considerably faster than in the central region and may go over into the "escape" regime. Hence, a plasma configuration with a monoenergetic beam of electrons develops exciting surface waves in a resonance manner. The development of such instabilities can significantly change the equilibrium configuration of the plasma. One of the possible methods of stabilization of unstable surface waves by a high-frequency field is proposed in this work.

The author expresses gratitude to K. N. Stepanov for helpful discussions of the results.

LITERATURE CITED

1. A. A. Ivanov, "Interaction of high-frequency fields with plasma," in: Problems of Plasma Theory [in Russian], No. 6, Atomizdat, Moscow (1972).
2. Yu. M. Aliev and V. P. Silin, "Theory of oscillations of plasma located in a high-frequency electric field," Zh. Éksp. Teor. Fiz., 48, 901 (1965).
3. V. V. Demchenko and A. Ya. Omel'chenko, "The problem of parametric resonance in a cold inhomogeneous isotropic plasma," Izv. Vyssh. Uchebn. Zaved., Radiofiz., 19, 471 (1976).

LIMITING CURRENT OF A CONICAL BEAM

A. S. Chikhachev

UDC 533.951

The study of the motion of a charged particle (electron) in the magnetic field of an electron beam is of great practical and theoretical interest. For example, Alfvén [1] studied the motion of a test electron in the field of a cylindrical beam and showed that there is a critical Alfvén current in a neutral beam. For currents above the limiting value [$I_A = (mc^3/e)\gamma\beta \simeq 17\gamma\beta$ kA] an electron can reach a certain point and turn back and not move along the beam.

We investigate the motion of a particle in the magnetic field of a conical relativistic electron beam to determine the effect of divergence on the existence of a limiting current. There is no electric field in the beam since it is assumed that there is a compensating background of positive stationary ions.

The equation for the magnetic field of a conical beam emitted from a center is

$$\operatorname{rot}\mathbf{H} = (4\pi/c)\mathbf{j},$$

where $\mathbf{j} = I_0\mathbf{r}/r^3$ and $I_0 = \text{const}$. Clearly the field can be described by a single component $\mathbf{H} = H_\varphi\mathbf{e}_\varphi$, where $\mathbf{e}_\varphi$ is a unit vector in the direction of variation of the azimuthal angle φ of a spherical coordinate system r, θ, φ. In this case

$$H = H_\varphi|_{\theta\leqslant\theta_0} = \frac{4\pi I_0}{cr}\operatorname{tg}\frac{\theta}{2}. \tag{1}$$

It is assumed that there is a current only for $\theta \leq \theta_0$. Outside the cone the magnetic field is

---

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 27-30, September-October, 1977. Original article submitted September 16, 1976.

0021-8944/77/1805-0611$07.50

$$H_\varphi|_{\theta>\theta_0} = \frac{4\pi I_0}{cr} \frac{\operatorname{tg}\frac{\theta_0}{2}}{\sin\theta} \sin\theta_0.$$

From now on, however, we consider the motion of an electron inside the beam only, and Eq. (1) is sufficient.

In the general case the solution of the equation of motion with the field (1) is a rather complicated problem. Alfvén [1] solved these equations for pure meridional motions, i.e., for $\dot{\varphi}(t) \equiv 0$. Even with this simplification it appears to be impossible to obtain an analytic solution for a conical current.

The equations of motion of a relativistic particle in spherical coordinates for $\dot{\varphi} = 0$ have the form

$$\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= \frac{eH}{\gamma mc} r\dot{\theta} = \frac{4\pi e I_0}{\gamma mc^2} \dot{\theta} \operatorname{tg}\frac{\theta}{2}, \\ r\ddot{\theta} + 2\dot{r}\dot{\theta} &= -\frac{eH}{\gamma mc}\dot{r} = -\frac{4\pi e I_0}{\gamma mc^2}\frac{\dot{r}}{r}\operatorname{tg}\frac{\theta}{2}, \end{aligned} \tag{2}$$

where γ is the ratio of the total energy of the particle to its rest energy. The equation of the trajectory can be obtained from (2)

$$r^2\theta'' + 2r\theta' + r^3\theta'^3 = -a(1 + r^2\theta'^2)\operatorname{tg}\frac{\theta}{2}, \tag{3}$$

where $\theta'(r) = d\theta/dr$, $a = 4\pi e I_0/\gamma mc^3$.

To within unimportant factors the quantity a is the ratio of the total beam current I_0 to the Alfvén current $I_A = mc^3\gamma\beta/e$. If we replace r in Eq. (3) by a new variable $\xi = \ln(r/r_0)$, where r_0 is an arbitrary scale factor, and linearize this equation by assuming $\theta \ll 1$ and $d\theta/d\xi \ll 1$, we obtain the relation

$$d^2\theta/d\xi^2 + d\theta/d\xi + (a/2)\theta = 0,$$

whose solution has an essentially different character depending on the sign of $2a - 1$.

In fact,

$$\theta(\xi) = \begin{cases} \left(C_1 e^{\frac{\xi}{2}\sqrt{1-2a}} + C_2 e^{-\frac{\xi}{2}\sqrt{1-2a}}\right) e^{-\frac{\xi}{2}}, & a < \frac{1}{2}, \\ \left(C_1 \cos\left(\xi\sqrt{2a-1}\right) + C_2 \operatorname{lin}\left(\xi\sqrt{2a-1}\right)\right) e^{-\frac{\xi}{2}}, & a > \frac{1}{2}. \end{cases} \tag{4}$$

Here C_1 and C_2 are arbitrary constants.

It is clear from Eq. (4) that for $0 < a < \frac{1}{2}$ the solution decreases as a power of r. For $a > \frac{1}{2}$, $\theta(\xi)$ oscillates, but in this case the beam is not suppressed. This is related to the condition $d\theta/d\xi \ll 1$. We note that for this condition to hold it is necessary that $\theta(2a-1)^{1/2} \ll 1$.

It is convenient to transform (3) to a first-order equation. By setting $p(\theta) = d\theta/d\xi$, we obtain

$$p\frac{dp}{d\theta} + p(1 + p^2) + a\operatorname{tg}\frac{\theta}{2}(1 + p^2)^{3/2} = 0.$$

For large enough values of a and not too small angles θ it can be assumed that $p \ll a \tan(\theta/2)(1 + p^2)^{3/2}$. Then the equation is considerably simplified:

$$p\frac{dp}{d\theta} + a\operatorname{tg}\frac{\theta}{2}(1 + p^2)^{3/2} = 0.$$

Hence, setting $p_0 = d\theta/d\xi|_{\xi=0}$, we obtain

$$p(\theta) = \left(\frac{1}{\left(1 - 2a\left(\ln\cos\frac{\theta}{2} - \ln\cos\frac{\theta_0}{2}\right)\right)^2} - 1\right)^{1/2}, \tag{5}$$

from which, obviously, $p = \infty$ for $\cos(\theta/2) = \cos(\theta_0/2)e^{1/\sqrt{2a}}$. This means that there is a point (r, θ) $(0 < \theta < \theta_0)$ on the electron trajectory at which the velocity makes a right angle with the radius-vector. A further increase in the angle between the velocity and the radius-vector to the particle indicates that it is moving opposite to the flow of beam electrons.

It is interesting to consider the case $\alpha < 0$ which corresponds to a converging electron beam. It is clear from (4) that the solution increases exponentially; i.e., if sufficiently large values of ξ are possible, the beam current must be considered suppressed for all values of $\alpha < 0$. It follows from (5) that there is no turning point of the trajectory of a beam electron. Evidently, for $\alpha < 0$ the nonlinear nature of the equation of motion has a stabilizing effect, but an exact analysis requires a numerical solution of Eq. (3).

Thus, an analysis of the motion of a test electron in a neutralized conical electron beam shows first that there is a range of values of the parameter α and the initial angular divergence of the beam for which there is no motion outside the limits initially set for the beam. In this case two types of motion are possible. The first possibility is that an initial small angular deflection is decreased as $r \to \infty$ ($\alpha < 1/2$). Evidently, this implies a certain self-focusing of the beam (with a possibility of its becoming cylindrical), which must lead to an oscillatory motion of the electrons. The second possibility ($\alpha > 1/2$) corresponds to an oscillatory motion of the electrons. This does not imply suppression of the beam current.

Finally, there is a range of values of α and θ_0 for which the beam current clearly will be suppressed. This range can be defined in the following way: $\alpha \tan(\theta_0/2) > 1$.

The author thanks A. V. Zharinov for helpful discussions.

LITERATURE CITED

1. H. Alfvén, "On the motion of cosmic rays in interstellar space." Phys. Rev., 55, 425 (1939).

EXPERIMENTAL VERIFICATION OF RADIATION FROM COMPTON ELECTRON CURRENTS

G. M. Gandel'man, V. V. Ivanov, Yu. A. Medvedev,
B. M. Stepanov, and G. V. Fedorovich

UDC 538.561

§1. Theoretical investigations of the electromagnetic radiation generated by currents of Compton electrons, formed in air near an impulse γ-ray source. have been performed in many studies (for example, [1-3]). On the other hand, much less attention has been paid to comparison of theory with experimental data, evidently because at first glance such a comparison reveals significant divergence between theory and experiment as to the form and duration of the signals generated. Thus, the theoretically calculated pulse has two semiperiods with duration of several μsec, while experiment [4] has recorded a three-semiperiod pulse with duration of tens and hundreds of μsec. There is a similar discrepancy in the ratios of field amplitudes in the different semiperiods; theory predicts a ratio of the order of decades, while experiment shows ratios near unity. A new approach to comparison of theory and experiment can be achieved if we assume that the total recorded electromagnetic impulse is in fact the sum of two signals of different nature. This may be confirmed by dividing the total signal into two components, with the parameters of one being close to those predicted for Compton radiation by theory, and the parameters of the other explicable by sufficiently general physical considerations.

To reliably confirm the conclusions of Compton electron radiation theory, it is desirable to establish some general property of the signal produced which is not related to detailed characteristics of the electron current, which vary in theoretical calculations depending on the assumptions made, and also vary from experiment to experiment. The presence of such a property in the experimental signal is then the criterion of the theory validity.

Since the published quantitative results on irradiated fields (see, for example [2, 5]) have been obtained by numerical integration of the Maxwell equations, while to detect general properties of signals it is more convenient to use analytic expressions for the fields, we will consider below the question of analytic description of the radiated signal characteristics. We will consider the signal connected with the disruption of symmetry of Compton electron signals upon their reaching the surface of the earth.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 30-34, September-October, 1977. Original article submitted August 5, 1976.

0021-8944/77/1805-0613$07.50